

Singularity avoidance in quantum-inspired inhomogeneous dust collapse

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In a previous paper, some of us studied general relativistic homogeneous gravitational collapses for dust and radiation, in which the density profile was replaced by an effective density justified by some quantum gravity models. It was found that the effective density introduces an effective pressure that becomes negative and dominant in the strong-field regime. With this set-up, the central singularity is replaced by a bounce, after which the cloud starts expanding. Motivated by the fact that in the classical case homogeneous and inhomogeneous collapse models have different properties, here we extend our previous work to the inhomogeneous case. As in the quantum-inspired homogeneous collapse model, the classical central singularity is replaced by a bounce, but the inhomogeneities strongly affect the structure of the bounce curve and of the trapped region.

PACS numbers: 04.20.Dw, 04.20.Jb, 04.70.Bw

Keywords: Gravitational collapse, black holes, naked singularity

I. INTRODUCTION

General relativistic gravitational collapses have been studied for many years since the pioneering work by Oppenheimer, Snyder and Datt [1] showed that a spherical matter cloud collapsing under its own weight leads to the formation of a black hole (BH). In this simple model, where the matter is described by homogeneous dust (i.e. pressureless) particles, the horizon forms at the boundary of the collapsing cloud before the formation of the central singularity. The system eventually settles to a Schwarzschild BH and the singularity remains inaccessible to far away observers. Since then, a lot of work has been done in order to understand the genericity and possible limitations of such a model. Singularity theorems by Hawking and Penrose [2] show that under reasonable requirements for the matter content (i.e. energy conditions) if trapped surfaces do form then a singularity must form as well. Still they do not provide any information about how and when these singularities form. Further investigations showed that for certain matter profiles that satisfy standard conditions the singularity can form at the same time of the formation of the trapped surfaces and can thus be visible to far away observers (see e.g. [3] and references therein for an overview of relativistic collapse). The two most important features that arise from the study of the complete gravitational collapse of a massive cloud within the theory of general relativity are the trapped surfaces and the singularity.

It is usually thought that the appearance of spacetime singularities is a symptom of the break down of classical general relativity, to be fixed by unknown quantum corrections. In Ref. [4], such a possibility was explored and the homogeneous collapse of dust and radiation was re-analyzed in the light of corrections that might arise in the strong field regime, as obtained within some Loop Quantum Gravity (LQG) approaches [5–7]. The procedure is similar to the one followed in models of Loop Quantum Cosmology (LQC) [8]. The main result obtained in [4] is that the singularity at the end of the

collapse is removed and replaced by a bounce. The expanding phase that follows the collapsing phase after the bounce affects the structure of trapped surfaces in the sense that the event horizon of the Schwarzschild spacetime does not form, being replaced by an apparent horizon that exists for a finite time. These results appear in accordance with other studies carried out along the same line in several contexts (see for example [9–14]).

In the case of the gravitational collapse of an astrophysical object such as a star, the homogeneous dust model is highly unrealistic. Here we attempt to extend the analysis developed in Ref. [4] to the more realistic case of inhomogeneous dust. Since already in the fully classical case the structure of trapped surfaces and singularity is drastically altered by the introduction of inhomogeneities, it is worth investigating what happens in the quantum-inspired model. The presence of inhomogeneities in the classical case allows for the central region in which the singularity forms to be visible to far away observers. This suggests that the structure of the bounce and of the trapped surface will be also altered in the quantum-inspired framework. We note that numerical studies of inhomogeneous gravitational collapse of scalar fields with LQG inspired corrections were reported in [15–17].

In cosmology, one may expect that the inhomogeneities arise from fluctuations at the quantum level of the gravitational field and the introduction of similar inhomogeneities in LQC models can be very difficult. Attempts to study inhomogeneous LQC models have been carried out by several authors [18–21]. On the other hand, when we deal with the collapse of a massive object such as a star, we start with a matter distribution where inhomogeneities can be described at a purely classical level. Therefore we can consider an initial configuration given by a classical inhomogeneous dust ball that collapses under its own weight and consider the quantum-gravity effects at a semiclassical level only toward the end of the collapse.

The paper is organized as follows. In Section II, we briefly review the formalism for the relativistic collapse of inhomogeneous dust matter. In Section III, we analyze how the relativistic picture is altered once quantum corrections in the strong field limit are considered. Finally, Section IV is devoted to a

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brief summary and discussion. In this paper, we use units in which $c = G_N = 1$ and absorb the factor 8π in the Einstein equations into the definition of the energy-momentum tensor.

II. CLASSICAL COLLAPSE

Here we assume that the collapse is spherically symmetric. Then the most general line element describing collapse in comoving coordinates can be written as

$$ds^2 = -e^{2\nu} dt^2 + \frac{R'^2}{G} dr^2 + R^2 d\Omega^2, \quad (1)$$

where $d\Omega^2$ represents the two-dimensional metric on the unit two-sphere. The metric functions $\nu(r, t)$, $G(r, t)$, and $R(r, t)$ are related to the physical density and pressures appearing in the energy-momentum tensor via the Einstein equations. The energy-momentum tensor in the comoving frame is diagonal and for a perfect fluid source depends only on density $\rho(r, t)$ and pressure $p(r, t)$. The Einstein equations can be written as

$$\rho = \frac{3M + rM'}{a^2(a + ra')}, \quad (2)$$

$$p = -\frac{\dot{M}}{a^2\dot{a}}, \quad (3)$$

$$\nu' = -\frac{p'}{\rho + p}, \quad (4)$$

$$\dot{G} = 2\frac{\nu' r \dot{a}}{a + ra'} G, \quad (5)$$

where we have absorbed the factor 8π into the definition of density and pressure. The scale factor $a(r, t)$ is a dimensionless quantity describing the rate of the collapse and is given by $R = ra$. The function $M(r, t)$ is related to the Misner-Sharp mass of the system $F = R(1 - g_{\mu\nu} \nabla^\mu R \nabla^\nu R)$ (describing the amount of matter enclosed within the shell labelled by r at the time t) via $F = r^3 M$ and is given by

$$M = a \left(\frac{1 - G}{r^2} + e^{-2\nu} \dot{a}^2 \right). \quad (6)$$

Given the freedom to specify the initial scale, we choose the initial time $t_i = 0$ such that $R(r, 0) = r$, which implies $a(r, 0) = 1$. Matching with an exterior Schwarzschild or Vaidya spacetime is done at the comoving radius r_b corresponding to the shrinking physical area-radius $R_b(t) = R(r_b, t)$ [22].

The addition of an equation of state for the matter content that relates p to ρ provides the further relation to close the system of the Einstein equations. If no equation of state is provided, one is left with the freedom to specify one free function, still satisfying basic requirements of regularity and energy conditions. One usually assumes that the matter content satisfies standard energy conditions (e.g. the weak energy conditions given by $\rho \geq 0$ and $\rho + p \geq 0$) and are regular and well behaved at the initial time at all radii. In this case, it is easy to prove that the singularity is reached for $a = 0$ and it is a strong shell-focusing curvature singularity,

where curvature invariants such as the Kretschmann scalar diverge. The curve $t_{ah}(r)$ that describes the apparent horizon is given by the condition $1 - F/R = 0$, which corresponds to $a(r, t_{ah}(r)) = r^2 M(r, t_{ah}(r))$, and it represents the time at which the shell labelled by r becomes trapped.

A. Homogeneous dust collapse

The simplest possible model that one can obtain from the above set of equations is that of homogeneous pressureless matter. From the condition $p = 0$, using Eq. (3) we get $M = M(r)$. From the requirement that the density is homogeneous, namely $\rho = \rho(t)$, we get $M = M_0 = \text{const.}$. Then Eq. (4) implies $\nu = \nu(t)$ and by a suitable reparametrization of the time we can set $\nu = 0$. This leads to $\dot{G} = 0$ in Eq. (5) from which we get $G = f(r)$. The Misner-Sharp mass in Eq. (6) can be written as an equation of motion and we see that homogeneity implies that $f(r) = 1 + kr^2$, with $k = \text{const.}$ The system is then fully solved once we integrate the equation of motion (6) written as

$$\dot{a} = -\sqrt{\frac{M_0}{a} + k}. \quad (7)$$

In the simple case of marginally bound collapse (corresponding to particles having zero initial velocity at radial infinity) given by $k = 0$, we obtain the solution for the scale factor

$$a(t) = \left(1 - \frac{3}{2} \sqrt{M_0} t \right)^{2/3}, \quad (8)$$

where the integration has been performed with the initial condition $a(0) = 1$. The singularity at the end of the collapse is simultaneous and occurs at the time $t_s = 2/3\sqrt{M_0}$, while the apparent horizon curve is given by $t_{ah} = t_s - 2r^3 M_0/3$. The horizon forms at the boundary of the cloud at the time $t_{ah}(r_b) < t_s$ and the singularity is therefore covered at any time.

B. Role of inhomogeneities

The introduction of perturbations in the classical density ρ is equivalent to consider a mass profile M that varies with r . Inhomogeneous models were first studied by Lemaitre, Tolman and Bondi [23]. From the Einstein equations, we obtain again $\nu = 0$ and $G = f(r) = 1 + r^2 b(r)$. The equation of motion becomes

$$\dot{a}(r, t) = -\sqrt{\frac{M(r)}{a(r, t)} + b(r)}, \quad (9)$$

and the scale factor for the marginally bound collapse case becomes

$$a(r, t) = \left(1 - \frac{3}{2} \sqrt{M(r)} t \right)^{2/3}. \quad (10)$$

Now the singularity is not simultaneous any more. The time at which the shell labelled by r becomes singular is given by the curve $t_s(r) = 2/3\sqrt{M(r)}$, while the apparent horizon curve is given by $t_{ah} = t_s(r) - 2r^3M(r)/3$. We now see that, depending on the behavior of the free function M , the structure of the singularity and of the apparent horizon curves can be very different. Given the continuity requirements that we must impose on M , it is reasonable to assume that close to the center the mass profile behaves like

$$M(r) = M_0 + M_2 r^2 + \dots \quad (11)$$

To have a physically viable model that describes a realistic object, we would expect that the density is radially decreasing outward. This implies that the parameter M_2 in Eq. (11) is negative.

In the inhomogeneous case, the singularity forms at $r = 0$ at the time $t_s(0) = 2/3\sqrt{M_0}$ and outer shells become singular at later times. The behavior of the apparent horizon near the center is also determined by the value of M_2 . For $M_2 < 0$, the apparent horizon forms at $r = 0$ at the same time of the formation of the singularity and the outer shells become trapped afterward. In this case, it is easy to prove that the central singularity can be visible, at least locally, to far away observers (meaning that there exist families of null geodesics escaping from the singularity). Also, given the nature of dust collapse (i.e. the absence of pressures), the boundary of the cloud can always be chosen at will thus making any locally naked singularity also globally naked [24–28].

Whether such naked singularities can practically affect observations in a realistic scenario is an entirely different matter. In fact, toward the last stages of collapse, if nothing happens to deviate from the classical relativistic picture, gravity dominates and densities are so high that for any practical purpose the radiation emitted from a collapsing object forming a naked singularity will be undistinguishable from that emitted from an object forming a BH [29]. On the other hand, if quantum effects were to modify the picture of collapse close to the formation of the singularity, the fact that such a region of the spacetime is not trapped behind an horizon might bear important implications for the future development of the cloud.

III. QUANTUM-INSPIRED COLLAPSE

The introduction of inhomogeneities in the classical dust collapse drastically alters the structure of the singularity and of the trapped surfaces. It is thus reasonable to ask whether inhomogeneities will play an important role even in our quantum-inspired model. There are different ways to introduce quantum corrections to the classical collapse in the strong field regime. Here we shall make use of a semiclassical treatment by assuming that the corrections to the Einstein tensor due to quantum effects can be taken into account by replacing the matter source by an effective matter source. Therefore we will write the usual Einstein equations in the following form

$$G_{\mu\nu} = T_{\mu\nu}^{\text{eff}}, \quad (12)$$

where $T_{\mu\nu}^{\text{eff}} \rightarrow T_{\mu\nu}$ in the weak field limit and $T_{\mu\nu}$ is the classical energy-momentum tensor for dust. The specific form of $T_{\mu\nu}^{\text{eff}}$ will depend on the specific approach to quantum gravity. Of course $\nabla_\mu T_{\text{eff}}^{\mu\nu} = 0$, but this is automatically satisfied in our approach because we will use the Einstein equations, which imply the Bianchi identity, and we will not overconstrain the theory by imposing specific requirements for the matter content. We just demand that the standard framework is recovered in the weak field limit and we will check *a posteriori* if a reasonable interpretation for the matter content is possible in the strong field regime. It is often believed that asymptotic freedom will play an important role at high densities in a way such that the gravitational interaction will diminish the density increases and infalling particles get closer. One way of modeling this behavior at a semiclassical level is to assume a variable coupling term G_N (that in the classical scenario is Newton's constant), where G_N will depend on ρ .

A similar approach is used to construct bouncing cosmological models within LQG. A homogeneous Friedmann-Robertson-Walker model is altered in such a way that the big bang singularity is replaced by a bounce [8]. In cosmological models, one expects that the large scale structures form from small inhomogeneities that are originated in the early Universe at a quantum level. Nevertheless, introducing inhomogeneities at a quantum level is not an easy task and there are difficulties due to the fact that we do not possess a viable theory of quantum gravity yet. On the other hand, for a collapse scenario, as already mentioned, the initial state of the system can be considered as purely classical and all the quantum corrections can be neglected at the initial time. The inhomogeneities that we consider are macroscopic perturbations in the matter distribution and appear in the stress energy tensor, where the classical ρ depends on r . We then follow the evolution of a classical inhomogeneous dust collapse to the point where quantum corrections become important and we treat these corrections at a semiclassical level modifying the stress energy tensor “shell by shell”. Following Ref. [4], we assume that the effective density can be written in the form

$$\rho^{\text{eff}} = \rho \left(1 - \frac{\rho}{\rho_{\text{cr}}} \right). \quad (13)$$

Here ρ_{cr} plays the role of a critical density associated with the minimum scale of collapse and can be related to the limit in which the gravitational attraction vanishes. The presence of the correction term in the effective energy density will induce an effective pressure in the dust collapse scenario that will become negative as the collapse approaches the critical stage. This effective pressure describes how the system approaches asymptotic safety. In the same manner, the mass function $M(r)$, which is related to the total Schwarzschild mass measured by far away observers $M_{\text{Sch}} = r_b^3 M(r_b)/2$, is replaced by a variable effective mass $M^{\text{eff}}(r, t)$ that decreases as the collapse progresses. Then following the standard matching conditions for classical general relativity one can perform the matching at the boundary with a radiating Vaidya exterior, which again has to be understood in the effective picture.

A. Homogeneous case

A model for homogeneous dust collapse inspired by the LQG corrections was investigated in Ref. [4] (see also Fig. 1). With the initial condition $a(0) = 1$, one finds the following solution for the scale factor

$$a(t) = \left[a_{\text{cr}}^3 + \left(\sqrt{1 - a_{\text{cr}}^3} - \frac{3\sqrt{M_0}}{2} t \right)^2 \right]^{1/3}, \quad (14)$$

where we have defined $a_{\text{cr}}^3 = 3M_0/\rho_{\text{cr}}$. It is easy to see that as the critical density goes to infinity we retrieve the classical homogeneous dust collapse model.

For the homogeneous semiclassical model, all the shells bounce at the same comoving time $t_{\text{cr}} = 2\sqrt{1 - a_{\text{cr}}^3}/(3\sqrt{M_0})$. Therefore, as a consequence of the homogeneity, we have a simultaneous bounce replacing the simultaneous singularity. The apparent horizon is again defined as the curve $t_{\text{ah}}(r)$ for which $a(r, t_{\text{ah}}(r)) = r^2 M^{\text{eff}}(r, t_{\text{ah}}(r))$. In the homogeneous case, one can see that the apparent horizon initially behaves like in the classical case, reaches a minimal radius r_* at the time $t_* = t_{\text{cr}}(1 - \sqrt{3a_{\text{cr}}^3}/\sqrt{1 - a_{\text{cr}}^3})$ and then re-expands crossing the boundary again before the time of the bounce. At the time of the bounce, we reach the asymptotic freedom regime in which the gravitational force vanishes. After t_{cr} , the cloud re-expands following a dynamics that is symmetrical to the collapsing case. Another trapped region forms in the expanding phase due to the fact that the gravitational attraction grows as the system leaves the asymptotic safe regime and eventually the whole cloud disperses to infinity.

B. Inhomogeneous case

An exact procedure to deal with inhomogeneities at the level of quantum gravity is presently not known. Luckily, for the purpose of studying a gravitational collapse we can consider a cloud that is already inhomogeneous in the weak field and thus begin with classical inhomogeneities as described in Section II B. The only guidelines we keep in mind when we introduce inhomogeneities are that we want to recover the classical case when the critical density goes to infinity and we want to recover the homogeneous case when the density perturbations go to zero. Nevertheless, even with these great simplifications, treating the problem analytically can prove to be too difficult. In what follows, we shall thus restrict our attention to the vicinity of the center of the cloud, by performing a Taylor expansion of all the relevant quantities near $r = 0$. This is possible due to the regularity of the functions involved even close to the classical singularity. We stress that in this way we are not assuming the existence of a bounce replacing the classical singularity. Indeed the same approach is used to study the formation of singularities in the classical case, where one can describe the collapse up to the critical time t_{cr} [3]. Here we use the same strategy and eventually we found a bounce: thanks to the regularity of the solution, *a posteriori* we can say that the model holds even after the bounce.

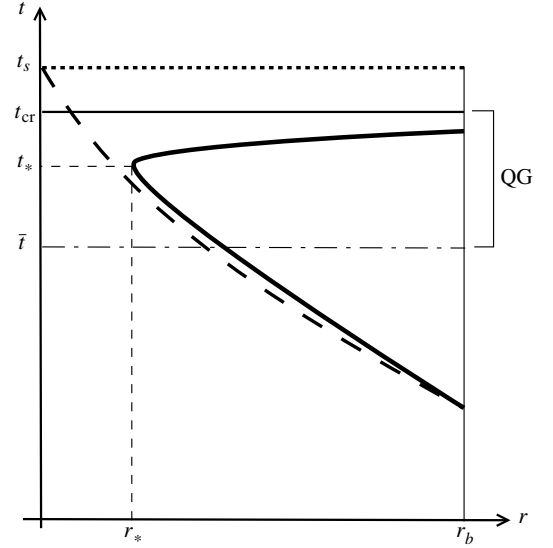


FIG. 1. Schematic illustration of the homogenous bounce in comoving coordinates. The collapse follows the classical model until the time \bar{t} at which the quantum gravity regime becomes important. The semiclassical apparent horizon (continuous thick line) separates from the classical one (dashed thick line), reaches a minimum r_* at the time t_* and then diverges. All the shells bounce at the same time t_{cr} before the classical time of the singularity t_s .

By expanding all the functions in the vicinity of $r = 0$, we are able to reduce the system of five coupled partial differential equations given by Eqs. (2)-(6) to a system of two coupled ordinary differential equations. Using Eq. (2) for the definition of the effective density in Eq. (13), we obtain the effective mass function $M^{\text{eff}}(r, t)$ that can be expanded in powers of r as

$$M^{\text{eff}}(r, t) = M_0^{\text{eff}}(t) + M_2^{\text{eff}}(t)r^2 + \dots, \quad (15)$$

with

$$M_0^{\text{eff}} = M_0 \left(1 - \frac{K}{a_0^3} \right), \quad (16)$$

$$M_2^{\text{eff}} = M_2 \left(1 - 2\frac{K}{a_0^3} \right) + 3M_0 \frac{K a_2}{a_0^4}, \quad (17)$$

where we have defined $K = 3M_0/\rho_{\text{cr}}$ and expanded the scale factor as

$$a(r, t) = a_0(t) + a_2(t)r^2 + \dots. \quad (18)$$

In order to write the equation of motion for the scale factor up to the second order, we need now to solve the full system of the Einstein equations in the effective picture. The dependence on t of the effective mass function will induce the presence of a non-vanishing effective pressure that can be expanded as $p^{\text{eff}} = p_0^{\text{eff}} + p_2^{\text{eff}}r^2 + \dots$, where

$$p_0^{\text{eff}} = -\frac{3M_0K}{a_0^6}, \quad (19)$$

$$p_2^{\text{eff}} = -\frac{6M_2K}{a_0^6} + \frac{18M_0K a_2}{a_0^7}. \quad (20)$$

From the remaining Eqs. (4) and (5) we get

$$\nu = \nu_2 r^2 + \dots = -\frac{p_2^{\text{eff}}}{\rho_0^{\text{eff}} + p_0^{\text{eff}}} r^2 + \dots, \quad (21)$$

$$G = b(r)e^{2A}, \quad (22)$$

with A defined by

$$\dot{A} := \nu' \frac{r\dot{a}}{a + ra'} = \dot{A}_2 r^2 + \dots \quad (23)$$

If we restrict ourselves to the marginally bound case given by $b = 1$, we can expand G as $G = 1 + 2A_2 r^2 + \dots$ and we obtain

$$\nu_2 = \frac{2K}{a_0^3} \frac{\frac{M_2}{M_0} - \frac{3a_2}{a_0}}{1 - \frac{2K}{a_0^3}}, \quad (24)$$

$$A_2 = 2 \int_0^t \nu_2 \frac{\dot{a}_0}{a_0} d\bar{t}. \quad (25)$$

Assuming that higher order terms are negligible, we finally get the expansion of the equation of motion (6) written order by order in the effective picture as

$$M_0^{\text{eff}} = a_0(-2A_2 + \dot{a}_0^2), \quad (26)$$

$$M_2^{\text{eff}} = a_2(-2A_2 + \dot{a}_0^2) + 2a_0[\dot{a}_0\dot{a}_2 - \nu_2\dot{a}_0^2]. \quad (27)$$

In the limit for $K = 0$ (corresponding to ρ_{cr} going to infinity), we retrieve the classical inhomogeneous collapse model, while in the limit for $M_2 = 0$ we obtain the homogeneous quantum-inspired model discussed in Ref. [4]. When we combine the above equations with Eqs. (16) and (17), we get the two equations of motion that need to be solved in order to obtain the expansion of the scale factor in the inhomogeneous quantum-inspired model. From the first one we get

$$\dot{a}_0^2 = \frac{M_0}{a_0} \left(1 - \frac{K}{a_0^3}\right) + 2A_2, \quad (28)$$

which, after we derive again with respect to t and substitute for \dot{A}_2 gives

$$\ddot{a}_0 = -\frac{M_0}{2a_0^2} + \frac{2M_0K}{a_0^5} + \frac{4K}{a_0^4 - 2Ka_0} \left(\frac{M_2}{M_0} - \frac{3a_2}{a_0}\right). \quad (29)$$

Then the second one leads to

$$\dot{a}_2 = \frac{M_2}{2a_0\dot{a}_0} \left(1 - \frac{2K}{a_0^3}\right) - \frac{M_0a_2}{2a_0^2\dot{a}_0} \left(1 - \frac{4K}{a_0^3}\right) + \nu_2\dot{a}_0. \quad (30)$$

Notice that in the inhomogeneous case the scale factor at zero order given by a_0 , as the solution of Eq. (28), is different from a in the homogeneous case. This is due to the non linearity of the Einstein equations that adds the term $2A_2$ in Eq. (28), which vanishes in the homogeneous limit. This is reflected in a different time of the bounce for the central shell with respect to t_{cr} in the homogeneous case, provided that the system is normalized with the same scaling at the initial time (see Fig. 2).

Our analysis is valid in the limit of small r , for which we can assume that all the higher order terms are negligible. In

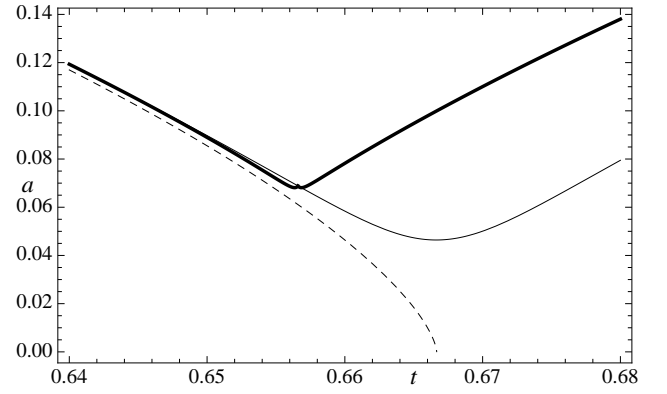


FIG. 2. The scale factor in the classical case (dashed line), in the homogeneous semiclassical case (thin line) and in the inhomogeneous semiclassical case for a fixed small value of r (thick line, $r = 0.01$). The following numerical values have been chosen: $M_0 = 1$, $M_2 = -0.1$, and $3M_0/\rho_{\text{cr}} = 0.0001$.

the general case, M_0 sets the scale for the collapse scenario, and this approximation breaks down at a certain radius for any given value of M_2 and ρ_{cr} . Classically, the limit of validity of the small r approximation is determined by M_2 only. Another important issue concerns the possibility of shell crossing singularities. The latter are weak curvature singularities that arise when different collapsing shells overlap [30]. They are obtained from the curvature scalars when the condition $a + ra' = 0$ is satisfied, but they do not signal geodesic incompleteness of the spacetime, which can be indeed extended through them. Shell crossing singularities do not appear in the case of the classical dust collapse if the energy density profile is homogeneous or radially decreasing outward. Nevertheless, for other density profiles and whenever pressures are present in the cloud one needs to check that no shell crossing singularities occur during the collapse. In the quantum-inspired scenario, in general the situation is made even more complicated by the fact that reflected shells will lead to caustics when overlapping with infalling shells, and these will also be indicated by shell crossing singularities. However, in the model studied here the bounce occurs first at the outer shells and thus if shell crossing singularities do happen they are confined outside the regime of validity of our “small r ” approximation.

One final point to mention concerns the classical physical density. We have considered here a classical density given by an expansion where ρ satisfies the Einstein field equations. In general, it is possible that the classical relativistic expression for ρ will not hold as the density approaches the critical value. This, in turn, will affect the form of the effective density derived in the semiclassical scenario. Since one does not know in principle how to write the modified density, and since we know that for ρ_{cr} going to infinity we must recover the classical case satisfying classical field equations, it makes sense to consider $\rho = \rho_{\text{GR}} + \epsilon(t)$, where ρ_{GR} is the relativistic energy density given by Eq. (2) and $\epsilon(t)$ is an arbitrary function that depends on ρ_{cr} and accounts for such modifications. The form of ϵ will then depend on the specific approach to the inhomogeneous case.

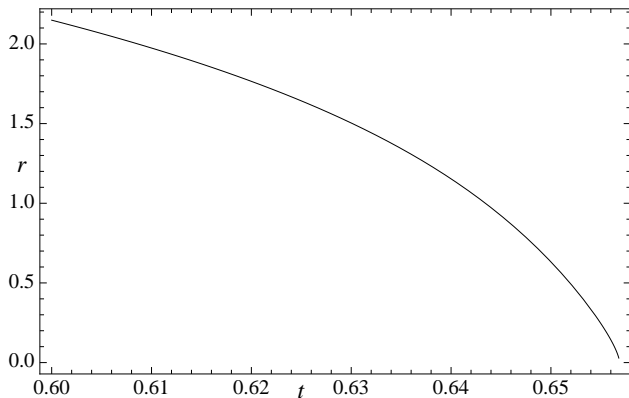


FIG. 3. The bounce curve $r_{\text{cr}}(t)$ in the inhomogeneous case. The following numerical values have been chosen: $M_0 = 1$, $M_2 = -0.1$, and $3M_0/\rho_{\text{cr}} = 0.0001$.

geneous system in quantum gravity. Nevertheless, ϵ must be negligible in the weak field regime (i.e. close to the initial time) and must go to zero as the critical density goes to infinity. It seems therefore reasonable to assume at first instance that the effect of $\epsilon(t)$ is negligible at any time. For this reason, and for simplicity, in order to minimize the number of free parameters in the analysis we have chosen to take $\epsilon = 0$ during the whole dynamical evolution. Solving the coupled system of equations given by (28) and (30) analytically could prove to be impossible but we can still understand the behavior of collapse near the center by solving the system of equations numerically.

C. Bounce and trapped surfaces

Because of the presence of inhomogeneities, the behavior of the collapsing cloud is affected “shell by shell”, and it is easy to see that the time of the bounce is different for every shell. We can define the “bounce curve” $t_{\text{cr}}(r)$ from the bounce condition

$$\dot{a}(r, t_{\text{cr}}(r)) = 0. \quad (31)$$

The crucial element that distinguishes the bounce from the homogeneous case is that $t_{\text{cr}}(r)$ (or inversely $r_{\text{cr}}(t)$) is not a constant (see Fig. 3).

This means that the asymptotic freedom regime is achieved at different times for each shell and thus the gravitational attraction does not vanish entirely at a specific time. An important consequence of the presence of the bounce curve is that the outer shells (still considering only shells close to the center) bounce before the inner shells, as opposed to the classical scenario where the singularity forms initially at the center when $M_2 < 0$. Then shell crossing singularities are not present near the center of the cloud. Nevertheless, there will be a certain radius at which the approximation for small r ceases to be valid. Expanding shells coming from the bounce will intersect the outer shells that still follow classical collapse

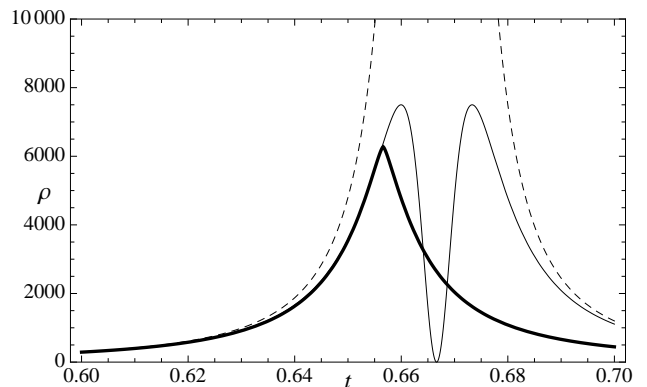


FIG. 4. Energy density in the classical scenario (dashed line), in the homogeneous semiclassical case (thin line) and in the inhomogeneous semiclassical case for fixed small value of r (thick line, $r = 0.01$). The following numerical values have been chosen: $M_0 = 1$, $M_2 = -0.1$, and $3M_0/\rho_{\text{cr}} = 0.0001$.

causing caustics, shell crossing singularities and a breakdown of the model.

The fact that t_{cr} is not constant implies also that the minimum value for the scale factor is different for every collapsing shell, and we thus have $a_{\text{cr}}(r)$ with the smallest value obtained for the central shell. A consequence of this fact is that the effective density does not vanish everywhere at a specific time, as opposed to the homogeneous case in which $\rho^{\text{eff}} = 0$ at the time of the bounce (see Fig. 4). Nevertheless, ρ^{eff} decreases as we approach the bounce and the effect of ρ_{cr} becomes more important than the effect of the inhomogeneity M_2 in this limit. This is reflected in the equations in the fact that the profile for the energy density goes from being decreasing radially close to the initial time to being increasing radially close to the time of the bounce.

Similarly to the classical inhomogeneous collapse model, the structure of formation of the trapped surfaces is given by the curve $t_{\text{ah}}(r)$ that represents the time at which the shell labelled by r becomes trapped. This is given implicitly by

$$a(r, t_{\text{ah}}(r)) = r^2 M^{\text{eff}}(r, t_{\text{ah}}(r)). \quad (32)$$

In the classical inhomogeneous case with $M_2 < 0$, the horizon forms initially at the time of formation of the singularity and then “propagates” outward meeting the event horizon at the boundary at a later time. Once we consider the semiclassical picture, the singularity is replaced by a bounce, the action of gravity is diminished approaching asymptotic freedom and the formation of trapped surfaces is delayed. The inverse of $t_{\text{ah}}(r)$ can be obtained from Eq. (32) by solving the quadratic equation

$$r^4 M_2^{\text{eff}} + r^2 (M_0^{\text{eff}} - a_2) - a_0 = 0. \quad (33)$$

It is easy to see that since a_0 has a minimum the apparent horizon will not pass through the shell $r = 0$ at any time. Therefore, like in the homogeneous case, we see that the apparent horizon behaves like the classical one in the weak field

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